



Norton-Thevenin Receptance Coupling (NTRC) as a Payload Analysis Tool

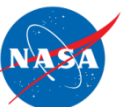
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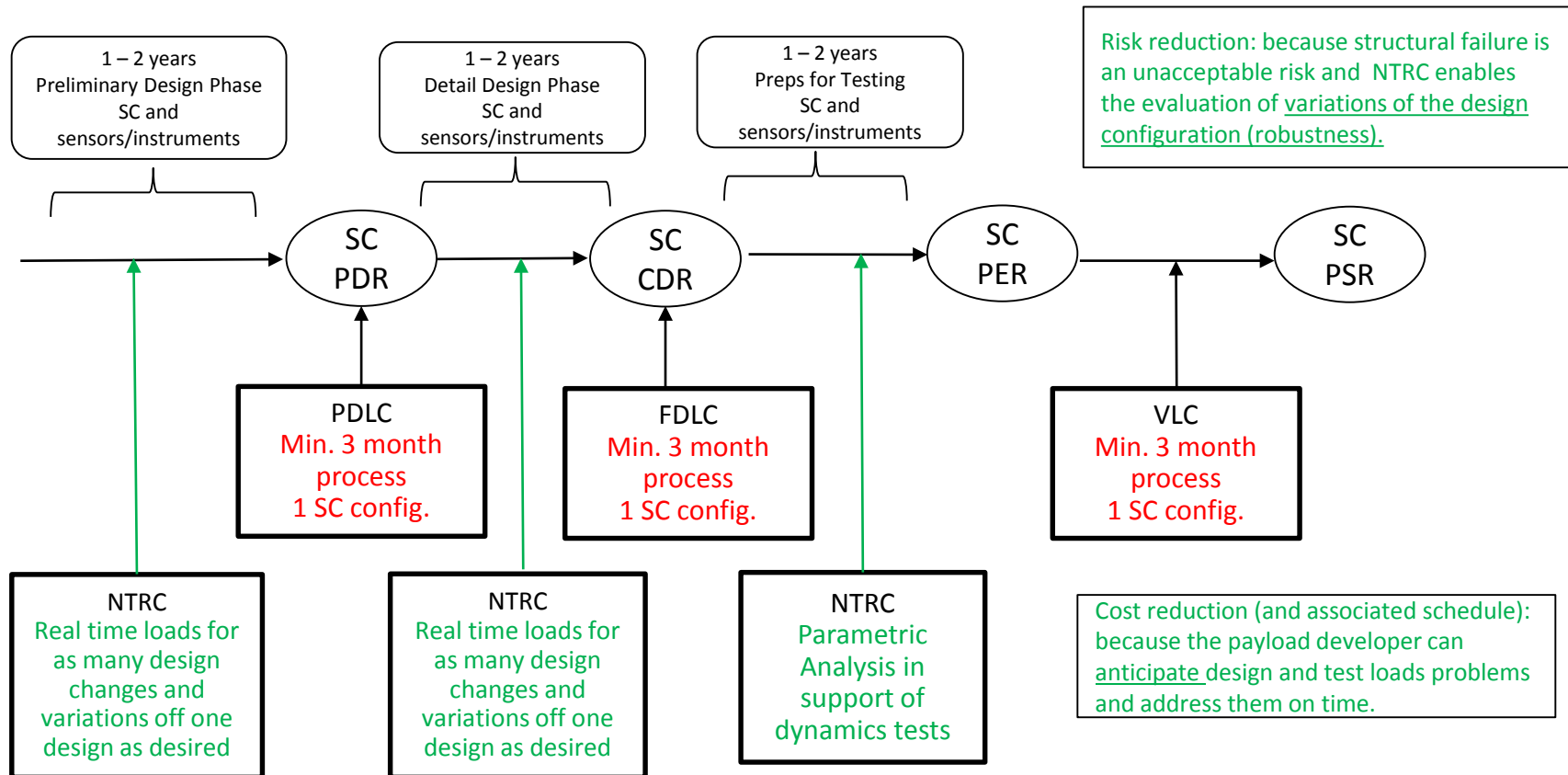
Agenda



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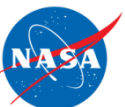
Typical Payload Development Process



Problem



- Booster/payload Coupled Loads Analysis (CLA) cycle schedule are not supportive of an effective and cost-efficient payload development process
- Payload development organizations traditionally can not run their own CLAs and need to wait long periods to get results and confirm design evolutions
- Payload development organizations need variational (parametric) CLAs (instead of single point CLA solutions) to gage response sensitivities and reduce risks
- These limitations impact payload development costs and schedules



Problem Statement



The problem that NTRC attempts to solve is the dependency of the payload organization to high CLA costs, long schedules, lack of standard capabilities to evaluate multiple configurations and unavailability of loads when needed.



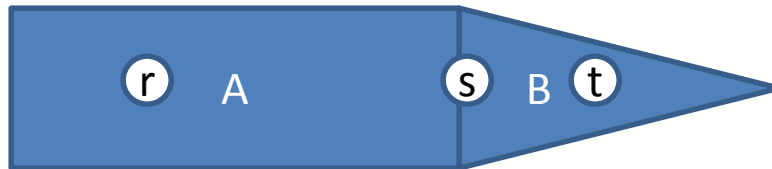
Solution Requirements



- **Accuracy:** Within +/-5% of traditional CLA results
- **System size:** minimal number of system DoFs to preserve accuracy and speed up computation (for parametrics)
- **Inputs:** Simplified minimal set of inputs: booster unloaded (i.e., no mass loading) interface accelerations (free-accelerations, FAs) and accelerance; payload interface accelerance
- **Solution Domain:** Solves in frequency domain (faster computation); avoid time-domain numerical integration



NTRC Methodology



Coupled System Accelerance [3]

$$\begin{bmatrix} A_{Cr} \\ A_{Cs} \\ A_{Ct} \end{bmatrix} = \begin{bmatrix} H_{Crr} & H_{Crs} & H_{Crt} \\ H_{Csr} & H_{Css} & H_{Cst} \\ H_{Ctr} & H_{Cts} & H_{Ctt} \end{bmatrix} \begin{bmatrix} F_{Cr} \\ F_{Cs} \\ F_{Ct} \end{bmatrix} \quad (1)$$

$$\text{CLA: } F_{Cs} = F_{Ct} = 0$$

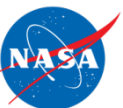
From (1) :

$$A_{Cs} = H_{Csr} F_{Cr} \quad (2)$$

$$A_{Ct} = H_{Ctr} F_{Cr} \quad (3)$$

C: coupled system (A+B)
 A: source with internal dofs r
 B: load with internal dofs t
 s: connecting dofs
 H: accelerance [g/lb]
 W: Impedance [lb/g] = H^{-1}
 F: [lb], A: [g]

H_{xyz} = Accelerance for
 System X with response at y
 dofs due to forces applied
 at z dofs





NTRC Methodology (Cont)

Receptance (Accelerance) Coupling for two substructures [3]:

$$\begin{bmatrix} H_{Crr} & H_{Crs} & H_{Crt} \\ H_{Csr} & H_{Css} & H_{Cst} \\ H_{Ctr} & H_{Cts} & H_{Ctt} \end{bmatrix} = \begin{bmatrix} H_{Arr} & H_{Ars} & 0 \\ H_{Asr} & H_{Ass} & 0 \\ 0 & 0 & H_{Btt} \end{bmatrix} - \begin{bmatrix} H_{Ars} \\ H_{Ass} \\ -H_{Bts} \end{bmatrix} [H_{Ass} + H_{Bss}]^{-1} \begin{bmatrix} H_{Ars} \\ H_{Ass} \\ -H_{Bts} \end{bmatrix}^T \quad (4)$$

From (4) we can define H_{Csr} and H_{Ctr} as:

$$H_{Csr} = \frac{H_{Bss} H_{Asr}}{H_{Ass} + H_{Bss}} \quad (5)$$

$$H_{ctr} = H_{bts} [H_{ass} + H_{bss}]^{-1} H_{asr} \quad (6)$$





NTRC Methodology (Cont)

Rewrite (2) using (5):

$$A_{Cs} = [H_{Csr}] F_{Cr} = \frac{H_{Bss} H_{Asr}}{H_{Ass} + H_{Bss}} F_{Cr} \quad (7)$$

Rewrite (3) using (6):

$$A_{Ct} = [H_{Ctr}] F_{Cr} = H_{Bts} [H_{Ass} + H_{Bss}]^{-1} H_{Asr} F_{Cr} \quad (8)$$

Combine (7) and (8):

$$A_{Ct} = H_{Bts} H_{Bss}^{-1} A_{Cs} \quad (9)$$

Introduce Norton-Thevenin [1] to relate the free acceleration (A_{As}) to the coupled acceleration at the boundary:

$$A_{Cs} = [H_{Ass}^{-1} + H_{Bss}^{-1}]^{-1} H_{Ass}^{-1} A_{As} \quad (10)$$

Combine (9) and (10) to get desired expression of coupled payload response (A_{Ct}) as a function of LV free acceleration (A_{As}):

$$A_{Ct} = H_{Bts} H_{Bss}^{-1} [H_{Ass}^{-1} + H_{Bss}^{-1}]^{-1} H_{Ass}^{-1} A_{As} \quad (11)$$





Time Domain Solution

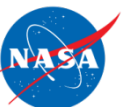
- From (11) we can define the NTRC transfer function between free acceleration and the desired load response quantity: TF

$$TF_{ts} = H_{bts} H_{bss}^{-1} [H_{ass}^{-1} + H_{bss}^{-1}]^{-1} H_{ass}^{-1}$$

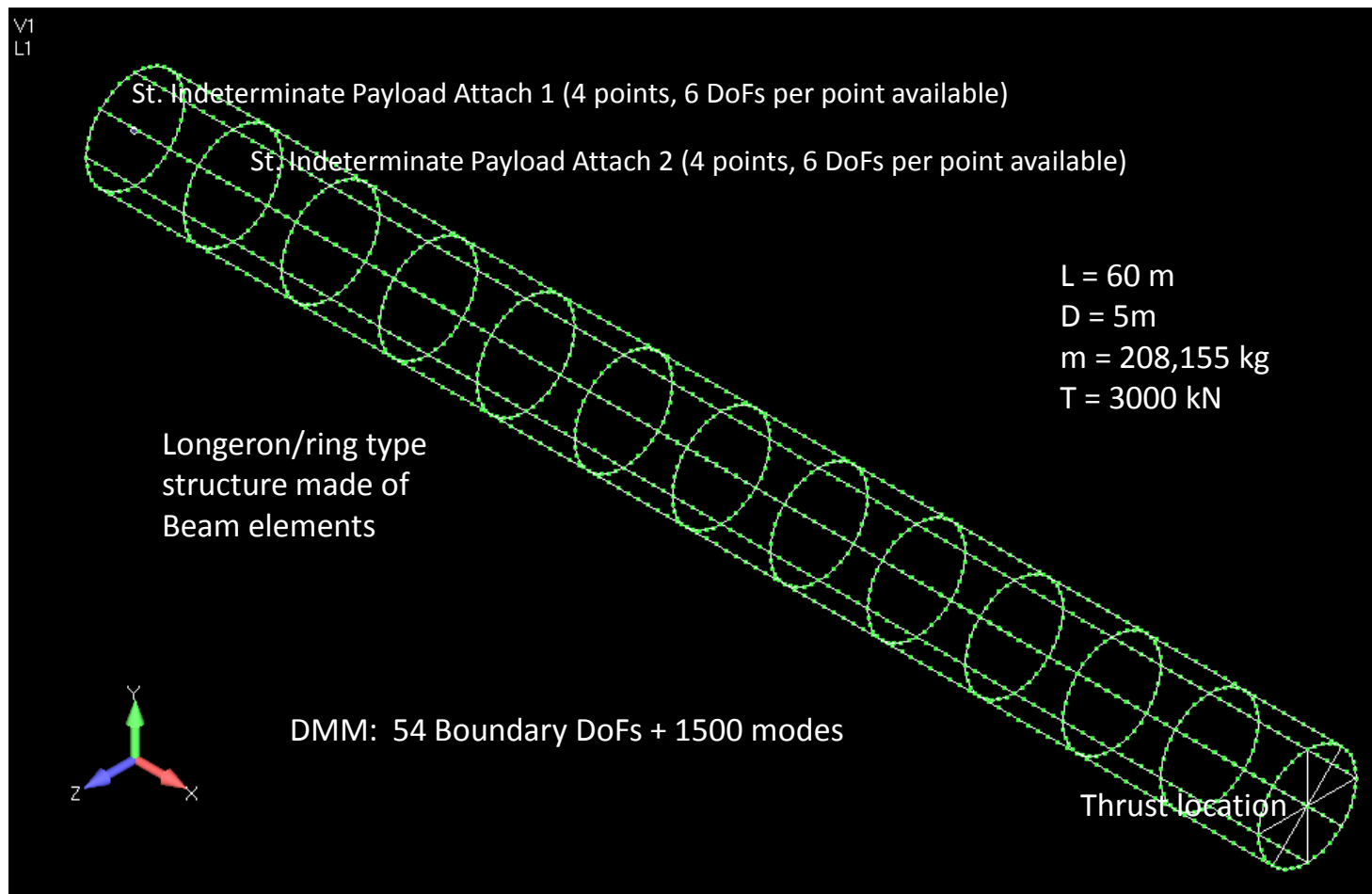
- We identified two methods to solve the time domain problem:
 - Multiplication in the frequency domain
 - Convolution
- Example of Multiplication in the frequency domain:
 - Perform transient analysis on LV to derive the free-acceleration (A_{As}) at payload interface
 - Transform A_{As} to frequency domain via FFT. Extract positive frequency terms and remove the $f=0$ Hz term (save for later)
 - Calculate accelerances (H) for payload and launch vehicle at common interface (consistent frequency range and delta-f). Come up with the NTRC TF and convert to FFT format.
 - Multiply to obtain (11)
 - Use IFFT to transform A_{Ct} back to the time domain (w/ $f=0$ term from FFT of A_{As})

Or

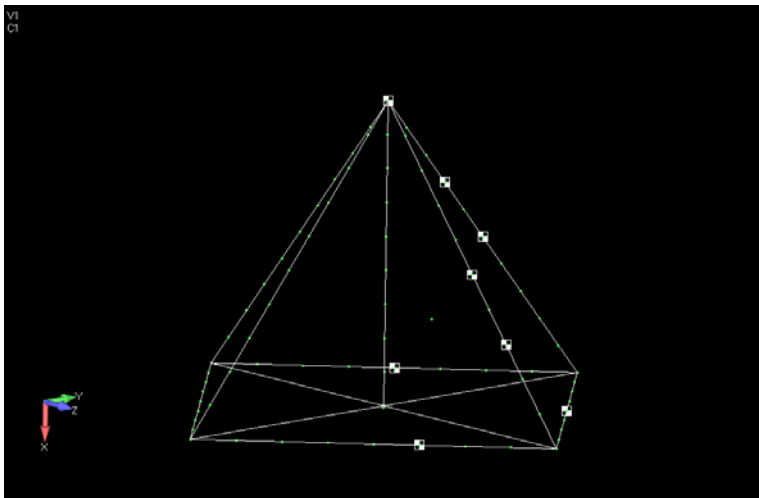
 - Obtain A_{Cs} from (10) and basedrive and PL with A_{Cs} to recover internal responses



Launch Vehicle FEM



Heavy Payload FEM



- Heavy payload FEM constructed to meet following requirements:
 - Weight: 3717 kg (8177 lbs)
 - Off-axis CoG
 - 1st lateral/rocking frequency 10-20 Hz (FEM: 10.6 Hz)
 - 1st axial frequency 20-40 Hz (FEM: 31.6 Hz)
 - All frequencies wrt st. det. constraints
- DMM: 24 physical DoFs + 200 modes
- Acceleration and Stress Transformation Matrices (ATM, STM) generated for internal response computations



Problem Size Comparison



CLA

- LV: 1554 DoFs
- Payload: 224 DoFs
- System = $1554 + 224 - 24 = 1754$ DoFs

NTRC

- LV: 24 DoFs
- Payload: 24 DoFs
- System size: 24 DoFs





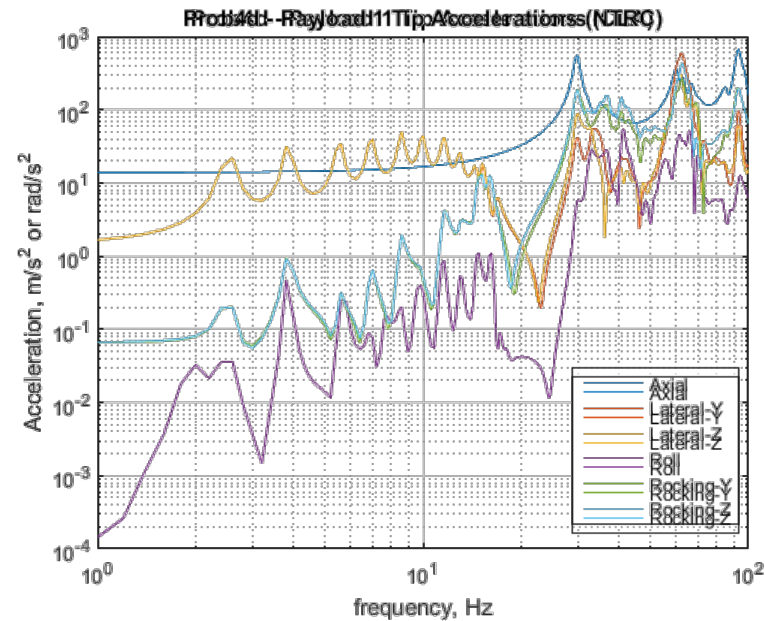
Frequency Domain Validation

- **Configuration:**
 - Heavy Payload 1: Statically Indeterminate Attach to LV (@ Location “1”)
 - 4 points, 6 DoFs per node = 24 DoFs attach
 - Heavy Payload 2: Statically Indeterminate Attach to LV (@ Location “2”)
 - 4 points, 3 Translational DoFs per node = 12 DoFs attach
 - LV: Axial Thrust + Lateral engine forces
- **Analysis parameters:**
 - Frequency range 1-100 Hz
 - Frequency increment 0.2 Hz
 - Axial Thrust= 3000 kN
 - Lateral Engine Forces = 5% Axial Thrust
 - LV: 2% Free Modes, Payload 1: 2% Free Modes, Payload 2: 5% Free Modes



Payload Tip Accelerations

Payload 1

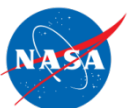


PAYLOAD ACCELERATIONS

Payload 1



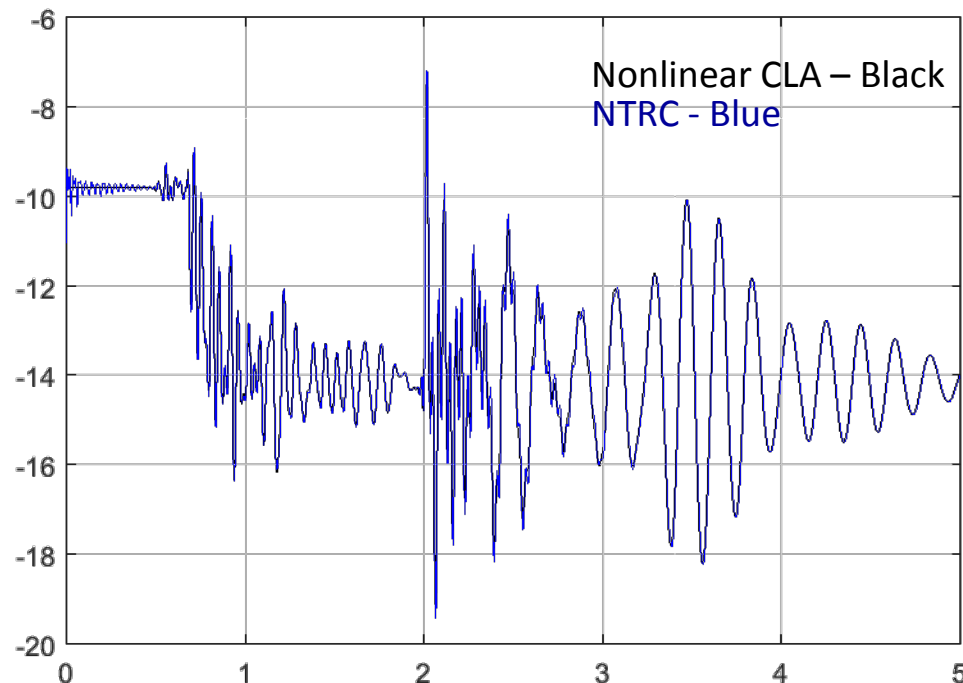
Prob4D - Peak Interface Acceleration (IMAGINARY Component)				
Item Description (m/s ² , rad/s ²)	CLA	NTRC	Abs Diff	% Diff
100001-X	670.544148238971	670.544148238970	1.0232E-12	0.0000%
100001-Y	57.599289908397	57.599289908398	-2.9843E-13	0.0000%
100001-Z	153.464513413116	153.464513413121	-5.0022E-12	0.0000%
100001-RX	147.856742908633	147.856742908632	1.0232E-12	0.0000%
100001-RY	417.404855308618	417.404855308617	9.6634E-13	0.0000%
100001-RZ	190.929521226388	190.929521226388	0.0000E+00	0.0000%
100023-X	597.662011426432	597.662011426432	0.0000E+00	0.0000%
100023-Y	145.585572001938	145.585572001943	-5.0022E-12	0.0000%
100023-Z	53.673849503604	53.673849503604	-9.9476E-14	0.0000%
100023-RX	141.675733019278	141.675733019278	0.0000E+00	0.0000%
100023-RY	56.708856599972	56.708856599972	3.9790E-13	0.0000%
100023-RZ	538.606792247222	538.606792247222	0.0000E+00	0.0000%
100045-X	644.876291078103	644.876291078102	1.0232E-12	0.0000%
100045-Y	61.021091650899	61.021091650899	3.9790E-13	0.0000%
100045-Z	154.888895839373	154.888895839378	-5.0022E-12	0.0000%
100045-RX	57.289287737808	57.289287737809	-1.0019E-12	0.0000%
100045-RY	344.650978558030	344.650978558030	0.0000E+00	0.0000%
100045-RZ	125.352614097016	125.352614097015	9.9476E-13	0.0000%
100067-X	713.011482007449	713.011482007451	-2.0464E-12	0.0000%
100067-Y	161.622692985640	161.622692985645	-5.0022E-12	0.0000%
100067-Z	70.000456586274	70.000456586274	1.9895E-13	0.0000%
100067-RX	29.669812393752	29.669812393751	1.9895E-13	0.0000%
100067-RY	126.925939400610	126.925939400610	0.0000E+00	0.0000%
100067-RZ	292.153745181615	292.153745181615	0.0000E+00	0.0000%



Time Domain Validation

- NTRC results captures all relevant characteristics of a transient CLA
- NTRC matches CLA w/o steady-state to < 5%
- Time domain NTRC with steady-state matches CLA < 5% for significant payload responses
- Source of differences
 - Convergence of time domain analysis
 - FFT/IFFT processing
- Will continue to refine time domain analysis for Q5 activities (SLS)

I/F Acceleration – DOF 10000001-1 (Thrust)





Concluding Remarks

- NTRC was benchmarked against a variety of CLA analysis configurations, parameters, and for 1000s of payload response items.
- NTRC is an alternate coupling approach that can be used to replicate a standard LV CLA
- NTRC developed as a design tool for payload community with the minimum information required from LV providers
- NTRC is exact for frequency domain analysis
- NTRC shows excellent agreement with results from time domain CLA.
- NTRC benchmarking to a “real-world” transient CLA has been established

